# Fundamentally, there are no relations

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1. Until Russell, almost all philosophers thought that fundamentally, there are no relations. Leibniz famously argued in his correspondance with Clarke that relations, if they existed, would be "in two subjects, with one leg in one, and the other in the other, which is contrary to the notion of accidents" and that "there is no denomination so extrinsic as not to have an intrinsic one for its foundation".

2. Though it certainly sounds anachronistic, this reductionist attitude towards relations has some intuitive plausibility. Although contemporary property theorists usually assume that what they say about properties easily generalises to relations, relations pose problems that do not arise (at least not as sharply) in the monadic special case.

1. properties, but not relations, may be wholly present in one particular;
2. relations, but not properties, seem to have parts;
3. relations, but not properties, may be partially exemplified;
4. relations, but not properties (pace vectors), may be directed

3. The traditional hostility to relations may have deeper roots: It is the business of philosophy to explain, to give a unifying and thereby illuminating account of variegated reality, thereby showing that the diversity and heterogeneity we observe is merely apparent.

4. In his *Principles of Mathematics* (§212) Russell criticised both monadism and monism. Monadism (defended, according to Russell, by Leibniz and Lotze) holds that any truth of the form "$aRb$" is equivalent to some truth of the form "$Fa \wedge Gb$", while monism (represented, for Russell, by Spinoza and Bradley) replaces "$aRb$" by some predication of the whole consisting of both relata taken together, "$H(ab)$".

5. Against monadism, Russell urges that relational properties cannot be understood except as involving relations. Against monism, he argues, that it is unable to distinguish the two directions characteristic of (binary) asymmetric relations other than by distinguishing the two parts of the whole by some other asymmetric relation, thus embarking on a regress.

6. Subsequent on his influential criticism of attempts to reduce relations to properties, many philosophers bought from Russell some highly simplified account of why 20th century logic and philosophy is superior to its predecessors. Broadly Aristotelian logic and metaphysics, it was said, were crippled by their inability to get to grip with relations. Frege then freed logic from its artificial limitation to monadic properties, discovered predicate logic and opened up the way to a more solidly ontological account of relations. [1]

7. In part, this story rests on indubitable logical fact. By Church's Theorem, monadic first-order predicate logic is decidable, while dyadic first-order predicate logic is not. The philosophical relevance

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[1] This is the view of John Bacon (1995: 37): "...it is one of the few unequivocal metaphysical lessons of modern logic that relations are indispensable to an account of the world. It's all very well to fantasize them as a "supervenient" free lunch; but save for ontological anorectics, the consequent inanition holds little charm, least of all in desert landscapes."

of this difference, however, is not easily assessed: Kripke (1962) showed that monadic modal logic is likewise undecidable, and Meyer (1968) showed the undecidability of monadic relevance logic. But even if it is granted that any full predicate logic is importantly different from its monadic fragment, this is a difference in expressive power between different kinds of predicates, a difference on the level of representations not between what they represent. It is not clear whether the difference in logical behaviour of predicates cuts any ontological ice.

8. We may acknowledge that it is useful, and even indispensable, to use relational predicates in a description of the world that aspires to completeness, that their inclusion increases the expressive power of our language and is necessary to account for the validity of inferences we intuitively recognise as valid. This is not to say, however, that we have to acknowledge relations as a separate and irreducible *ontological* category. We may very well hold relational *talk* is irrreducible, while still attempting an ontological reduction of relations.

9. Humberstone (1996) calls a binary relation $R$ *$\wedge$-representable* iff there are monadic predicates $F$ and $G$ such that, for all $x$ and $y$, $xRy$ iff $Fx \wedge Gy$. Humberstone (1984) has shown that a binary relation $R$ is $\wedge$-representable iff it satisfies the following condition of "forgetful transitivity":

(1) $\forall x, y, u, z((xRy \wedge uRz) \rightarrow xRz)$

As this obviously is not true of all dyadic relations, not all such relations are monadically representable. We cannot, therefore, do away with truly relational (more than one-place) predicates and thus monadism, in its simplest form, is false.

10. Relational predicates are ineliminable. This does not entail, however, that relations are ontologically basic. We have to accept as true truthbearers featuring relational predicates, but we do not automatically have to provide them with facts involving relations as truthmakers. By a 'relational fact', in the following, I will mean a fact that makes some truthbearer true in which occurs essentially a predicate of adicity higher than one. Defined in this way, "…the existence of relational facts does not automatically entail any real existence for relations" (Campbell 1990: 97). The question whether relations exist as an ultimate ontological category or whether they can be 'reduced' to properties of complexes is a matter of how to analyse relational facts.

11. Could we, e.g., 'reduce' relations to relational properties, properties of the form *standing in $R$ to $a$* for some relation $R$ and some particular $a$? Hochberg (1988: 196) has argued that the answer is no: relational properties do not give us expressive power enough to state even their own identity conditions. To say that, generally and as a matter of logical truth, if $a = b$, then $\lambda x(aRx) = \lambda x(bRx)$, we need to quantify over relations. The individuation of relational properties, then, presupposes a prior individuation of relations. Furthermore, a 'reduction' of relations to relational properties is in danger of being trivial: a thesis according to which the fact that $aRb$ is 'really' the fact that $(\lambda x(xRb))a \wedge (\lambda y(aRy))b$ but that does not substantiate any sense in which the latter is in some sense prior to the first is not of much interest.

12. How could such a priority be spelt out? Supervenience, nowadays, is the natural candidate. Supervenience allows us to say that relational vocabulary is ineliminable, even though its applicability "rest on and is exhausted by" monadic facts (Campbell 1990: 100). Supervenience, however, comes in different forms. Humberstone's result shows that we cannot expect a one-to-one correlation between binary and conjunctions of monadic predicates, so both weak and strong supervenience are ruled out. Should we then opt for global supervenience, some claim to the effect that no two worlds can differ in relational fact without differing in what monadic properties are exemplified? But this could be true even if relations were irreducible.

13. Distinguish different notions of 'internal relation': The relation $R$ obtaining between $a$ and $b$ is internal iff

1. it is essential to $a$ and $b$ that $Rab$ (Moore)
2. necessarily, if $a$ and $b$ exist then $Rab$ (Armstrong)
3. it holds between any duplicate of $a$ and any duplicate of $b$ (Lewis)
4. "$Rab$" is made true by truthmakers that 'involve' only intrinsic properties.

Could we 'reduce' relations by showing they are all internal? Two problems:

1. Internal relations are still relations.
2. There are external relations: spatio-temporal distance is accidential (hence not Moore-internal), causation is contingent (hence not Armstrong-internal), similarity may be in extrinsic respects (hence not Lewis-internal) and truthmaking is an external relation on any account (clearly so for predications of extrinsic properties; but also because truthbearers do not have their meaning intrinsically).

We cannot reduce external to internal relations if some fundamental properties are extrinsic. And some are: the property of the world to be all there is is an extrinsic (and non-relational) property of it and it also seems fundamental.

14. Relational facts are peculiar in many respects. A first problem concerns the relation between relations and relational properties. Necessarily, whenever a dyadic relation $R$ is exemplified, say by $a$ and $b$, two relational properties, ***having R to** $a$* and ***having R to** $b$*, are exemplified too, by $b$ and $a$ respectively. Even if it is a "dodge" and a "linguistic sleight of hand" to reduce relations to relational properties, the latter do exist if the former do. But what is the relation between a relation and its relational properties? If relations and relational properties are distinct, their co-exemplification tie constitutes a necessary connection between distinct existences, something which is ***prima facie*** mysterious and has to be explained or explained away. If relational properties are somewhat derived from, or contained in, their relation, the latter is complex.

15. A second problem concerns the relation between a relation and its relata. In the case of properties, we face a number of theoretical choices and could opt, for example, for a bundle theory, making the properties parts, constituents or moments of their bearer, explaining resemblance by overlap, sharing of constituents or exact similarity of moments. It seems very difficult to include relations in such a bundle: if we include relational properties (thereby distinguishing between the 'parts' of the relation in terms of the difference of the relata), we can no longer explain the resemblance between two pairs linked by the same relation in terms of overlap, sharing or similarity. If we opt to preserve this possibility, we make it mysterious how relations can be 'in' their relata.

16. A third problem concerns the direction component of relations – is it always, or only sometimes present? We face a double dilemma, for both symmetric and asymmetric relations: If $R$ and $R'$ are different if they have different senses, then the relational fact $aRb$ is different from the relational fact $bRa$ even if $R$ is necessarily symmetric. If they can be identical, even if their senses are different, then what distinguishes $aRb$ from $bRa$ for asymmetric relations $R$? If the 'sense' of a relation is something over and above the order of its constituents, then how can we identify $aRb$ with its converse $a\dot{R}b$ for necessarily symmetric $R$? If it just consists in this order, how can we distinguish $a\dot{R}b$ from $aRb$ for asymmetric relations? It just does not seem possible to hold both $a\dot{R}b = aRb = bRa$ for symmetric relations and $a\dot{R}b \neq aRb \neq bRa$ for asymmetric relations.

17. The most serious problem for relations, in my view, concerns the interaction between direction and order. I call "direction" of a relation it's from-$x$-to-$y$ feature: a relation and its converse differ in their direction. "Order" stands for the fact that relations apply to their arguments ***in a certain way***:

the relational complexes $Rab$ and $Rba$ differ in the order of the relation they contain. It is possible to have order without direction, but it is not possible to have direction without order. The argument (due to / inspired by Fine 2000) seems to show that nothing ontologically fundamental can have ***both*** order and direction. Because relations essentially have both, they cannot exist.

18. Let us suppose that Othello loves Desdemona, but that Desdemona does not love Othello. Let us distinguish:

$Rab$ Othello's loving Desdemona.

$Rba$ Desdemona's loving Othello.

$\dot{R}ab$ Othello's being loved by Desdemona.

$\dot{R}ba$ Desdemona's being loved by Othello.

The ontology of the situation imposes three constraints that cannot be simultaneously fulfilled:

$Rab = \dot{R}ba$ There is nothing else to Desdemona's being loved by Othello than what there is to Othello's loving Desdemona. We plainly have two descriptions of the same fact.

$Rab \neq Rba$ The two facts are different because one can obtain without the other.

$Rba \neq \dot{R}ba$ The two facts are different because one can obtain without the other.

This shows that nothing can be such that both order and direction are essential to it. Order forces us to distinguish $Rab$ and $Rba$. Direction forces us to distinguish $Rba$ and $\dot{R}ba$. But their interplay forces us to identify $Rab$ and $\dot{R}ba$. After all, converses are ***defined*** by

(2) $\dot{R}$ is the converse of $R \quad :\Longleftrightarrow \quad \forall x, y\ (\dot{R}xy \leftrightarrow Ryx)$

If we do not identify $Rab$ and $\dot{R}ba$, we either don't mean order by "order" or we don't mean direction by "direction".

19. The formulation of the problem in terms of identities and differences of relational facts makes it clear that the problem arises both for theories of universals and of tropes. For universals, the question may be put as follows: both their direction and the order in which they take their arguments seem essential to relation universals, but they cannot both be. For tropes, the question becomes: how can the relational tropes in $Rab$ and in $\dot{R}ba$ be exactly similar / of the same type / instances of the same universal, if they differ both in order and direction, and difference in order accounts for the dissimilarity (difference of type etc.) between the tropes in $Rab$ and in $Rba$ and difference in direction accounts for the dissimilarity between the tropes in $Rba$ and in $\dot{R}ba$.

20. One way out is adopting positionalism (Williamson 1985) and to say that the neutral amatory relation, e.g., comes with two extra entities, the argument-places Lover and Beloved, which it associates to its terms. Exemplification of the relation must then "be understood to be relative to an assignment of objects to argument-places" (Fine 2000: 11). But then how could there be strictly symmetrical relations, e.g. relations $R$ such that $a$'s being $R$-related to $b$ is the ***same relational fact*** than $b$'s being $R$-related to $a$? And what relates the arguments to the argument-places? We are embarking on another regress.

21. Fine's moral:

> "it is a fundamental fact [...] that relations are capable of giving rise to a diversity of completions in application to any given relata and there is no explanation of this diversity in terms of a difference in the way the completions are formed from the relation and its relata" (Fine 2000: 19)

Fine calls these relations "neutral" and explains differential applicability by relations being 'completed' by their relata in the same manner as in some exemplary relational fact: the amatory relation, e.g., holds between Don José and Carmen in the same way as it holds between Abelard and Eloise, but not in the same way as it fails to hold between Carmen and Don José.

22. The notion of co-mannered completion, however, is problematic. In so far as Fine's anti-positionalism is a kind of resemblance, paradigm-based nominalism, it shares its problems (cf. MacBride 2007). But it also has some problems of its own: Are we really speaking of Abelard and Eloise when we say that Don José loves Carmen? Fine suggests that we may understand it as involving a rigid reference to some manner of completion, which is the equivalence class of all co-mannered completions of the same relation, and to which reference is fixed by some exemplary relational fact. Part of the worry remains, however: When we say, of Carmen and Don José, that the latter loves the former but not vice versa, are we really saying that the amatory relation holds of them in some manner, but not in another? Would they love each other in both manners if they each loved the other? And if they loved each other, would they love each other in the same way than Abelard loves Eloise? The answer, as Fine (2000: 24, fn. 13) notes, is no, for otherwise Eloise would love Abelard. We thus have at least three ways two persons may love each other (three ways in which the amatory relation may be completed by two persons): from the first to the second, from the second to the first and reciprocally. For an $n$-place relation, there will be up to $2^n - 1$ ways it may be completed by its $n$ relata. If there is no upper bound to the adicity of asymmetrical relations and if the number of actual individuals is finite, we might run out of exemplars – and should we not still be able to say that some relation is completed by all the actual individuals in such-and-such a manner, though there are other, pairwise different manners in which it might have been completed?

23. My moral: neutral relations are not relations, but structural properties of wholes. They *intrinsically* structure their bearer, and not by relating their parts.

24. Let us call a *foundation* of a relation $R$ any property on which it supervenes. Josh Parsons (2003), defending the British idealists against Russell, has argued that relations supervene at least on structural properties of worlds, if not of anything smaller. The argument is simply that any relation holds between some things; because wholes inherit the truth-making properties of their parts, any fact that makes true the statement that the relation holds among these things also makes true a non-relational statement about something of which these things are parts. So every relation $R$ has a foundation, which we may denote by "$\bar{R}$". Supervenience on structural properties is not, however, enough to get rid of relations, for structural properties could still essentially involve relations.

25. From the foundation of a relation, we get its *adicity* with respect to a specific exemplification. *Contra* Armstrong, I do not take the adicity of a relation to be invariable: relations can hold between various numbers of relata. For any exemplification of a relation $R$, we get its adicity by successively reducing the size of the world of which its foundation $\bar{R}$ holds. If at any stage we get an object, $a$, which has $\bar{R}$ but lacks *has a proper* $\bar{R}$ *part*, we stop. It does not matter if our minimal supervenience base for $R$ is not unique or if it is gerrymandered; all we need is a principled way of counting its relevant parts. Suppose, then, that we have a binary relation $R$, holding between $a$ and $b$. I claim that $R$ is either internal or external, i.e. that it is intrinsic to $a \oplus b$. Suppose it is not. Then $\bar{R}$ is an extrinsic property

of either $a$, $b$ or $a \oplus b$. Then there is an extrinsic property of $a \oplus b$ on which $R$ supervenes.[2] But this means that $\bar{R}$, which we assumed to be a property of $a \oplus b$, is not a supervenience base for $R$ after all. The minimal supervenience base for $R$ will include more objects than just $a$ and $b$ and so we were wrong to assume that $R$ is binary.

26. Relations, then, supervene on intrinsic properties of the fusion of their relata. Such properties are compositional properties of the form *having parts standing in relation* $R$. Why is such a property intrinsic? Suppose $a$ has it. Then $a$ has parts $a_1, \ldots, a_n$ such that $R(a_1, \ldots, a_n)$. Any $a_i$ then has a relational property $F_i$ of the form $\lambda x \exists x_1, \ldots x_{n-1}(A_1 x_1 \wedge \ldots \wedge A_{i-1}x_{i-1} \wedge A_{i+1}x_{i+1} \wedge \ldots \wedge A_n x_n \wedge R(x_1, \ldots, x_{i-1}, x, x_{i+1}, \ldots, x_n)$. where "$A_i$" denotes $x_i$'s nature. Although *having an* $F_i$ *part* is an extrinsic property of $a$, the conjunction of all these properties which is *having parts standing in relation* $R$ is intrinsic.[3]

27. Internal and external relations differ with respect to the relational properties they give rise to. If $R$ is an internal relation holding between $a$ and $b$, the relational property $\lambda x(Rxb)$ of $a$ supervenes on the intrinsic properties of $a$ and $b$. If we let "$B$" denote $b$'s intrinsic nature, $\lambda x \exists y(Rxy \wedge By)$ thus is an intrinsic property of $a$. If $R$ is an external relation, the relational property of $a$ is then $\lambda x \exists y(Rxy \wedge By \wedge x \text{ is part of a } C)$, which is extrinsic. Internal, but not in general intrinsic, relations thus give rise to intrinsic relational properties of their relata. Internal relations, being nothing over and above the intrinsic properties of their relata on which they supervene, are not really 'relations' in the (for us) problematic of this term: they are not mysteriously 'between' anything, they are located where their terms are and they account for resemblances between different things.

28. What about external relations? Having a foundation, they supervene on *extrinsic* properties of their relata, e.g. *being part of an* $R$*-connected whole*. These properties are irreducibly structural, they are exemplified only by the whole containing all relata as parts. The question whether, fundamentally, there are relations now becomes: are these structural properties of wholes prior or posterior to the relations between the parts?

29. They are posterior. It is at least possible that there are structured simples. If it is possible that there are structured simples, structure is not always the result of relatedness. If structure does not always stem from relatedness, it never does: structure is prior.

30. In this way, it seems, we may have it both ways: we acknowledge the indispensability of relational talk and do not have to take relations with ontological seriousness. We have the expressive power without the ontological price.

[2] Proof: Suppose $\bar{R}$ is an extrinsic property of $a$. Then there is a duplicate $a'$ of $a$ that lacks $\bar{R}$. So $a' \oplus b$ is a duplicate of $a \oplus b$ that lacks *having an* $\bar{R}$ *part*. If $R$ supervenes on $\bar{R}$, then it also supervenes on this property of the whole.

[3] To see this, consider a binary relation $R$. I claim that "$\lambda x(\lambda y(Rxy)a)b$" and "$(\lambda x(Rxb))a \wedge (\lambda y(Ray))b$" ascribe the same property to $a$ and $b$, namely that $R$ holds between them. But "$\lambda x(\lambda y(Rxy)a)b \rightarrow ((\lambda x(Rxb))a \wedge (\lambda y(Ray))b)$ is just what is known as $\alpha$-conversion. Schoenfinkel observed already in 1924 that $\alpha$-conversion allows us to reduce functions of several variables to unary functions (cf. Barendregt 1981: 6, 22).